Z(3)-symmetric effective theory for pure gauge QCD at high temperature

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Outline

- Background
  - Dimensional reduction and QCD
  - The center symmetry and the Wilson line
- Construction of the new theory
  - Degrees of freedom, potentials
  - Perturbative matching to full theory
  - The Z(3) domain wall
- Phase diagram of the new theory
- Conclusions and future directions

QCD and dimensional reduction

- Conventional DR: at high $T \gg gT$, integrate out all non-static modes ($m \sim 2\pi T$) to obtain $3d$ effective theory for the static modes

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\mathcal{L}_{\mathrm{EQCD}} &= g_3^{-2}\Big\{\tfrac{1}{2}\,\mathrm{Tr}\,F_{ij}^2 + \mathrm{Tr}\big[(D_i A_0)^2\big] \\
&+ m_{\mathrm{E}}^2\,\mathrm{Tr}\,(A_0^2) + \lambda_{\mathrm{E}}\,\mathrm{Tr}\,(A_0^4)\Big\} + \delta\mathcal{L}_{\mathrm{E}}, \\
& \quad g_3 \equiv \sqrt{T}g,\ m_{\mathrm{E}} \sim gT,\ \lambda_{\mathrm{E}} \sim g^2
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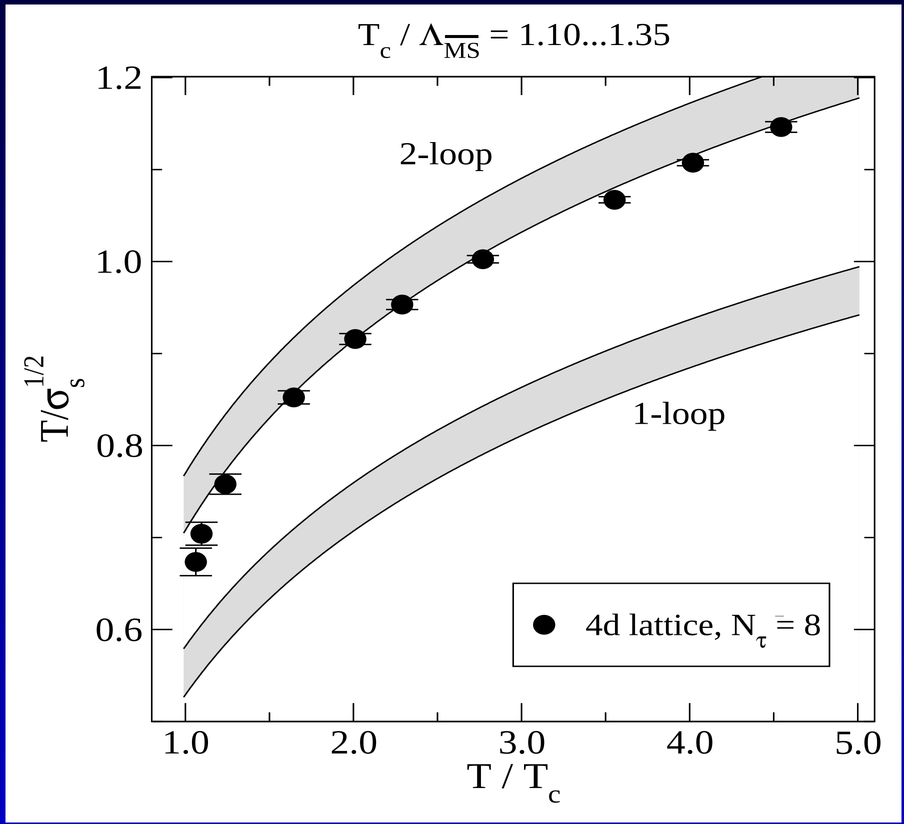
- New theory sufficient to describe equilibrium thermodynamics at length scales $\gtrsim 1/(gT)$
- Parameters available through comparison of long distance correlators in EQCD and full QCD

- Partial success: impressive perturbative results derived at high T



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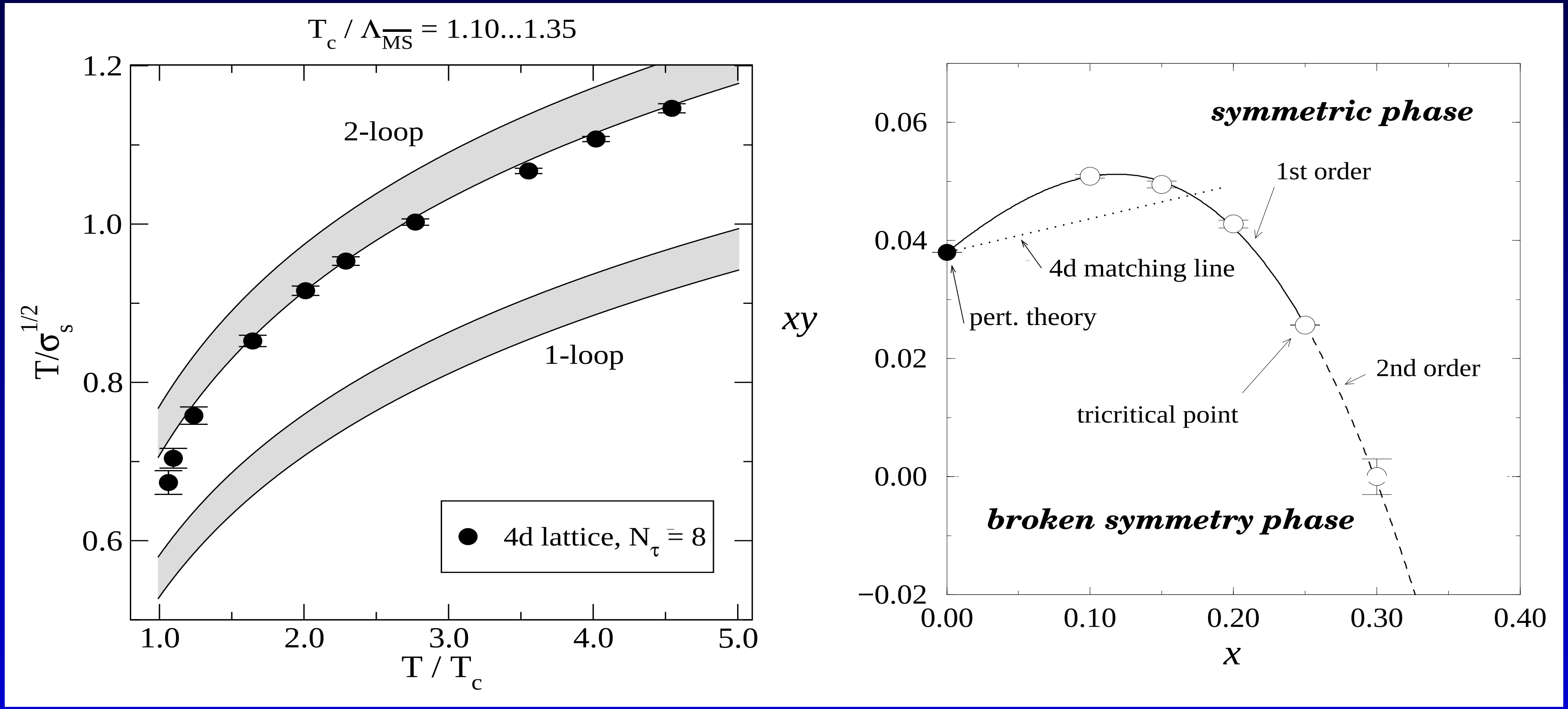

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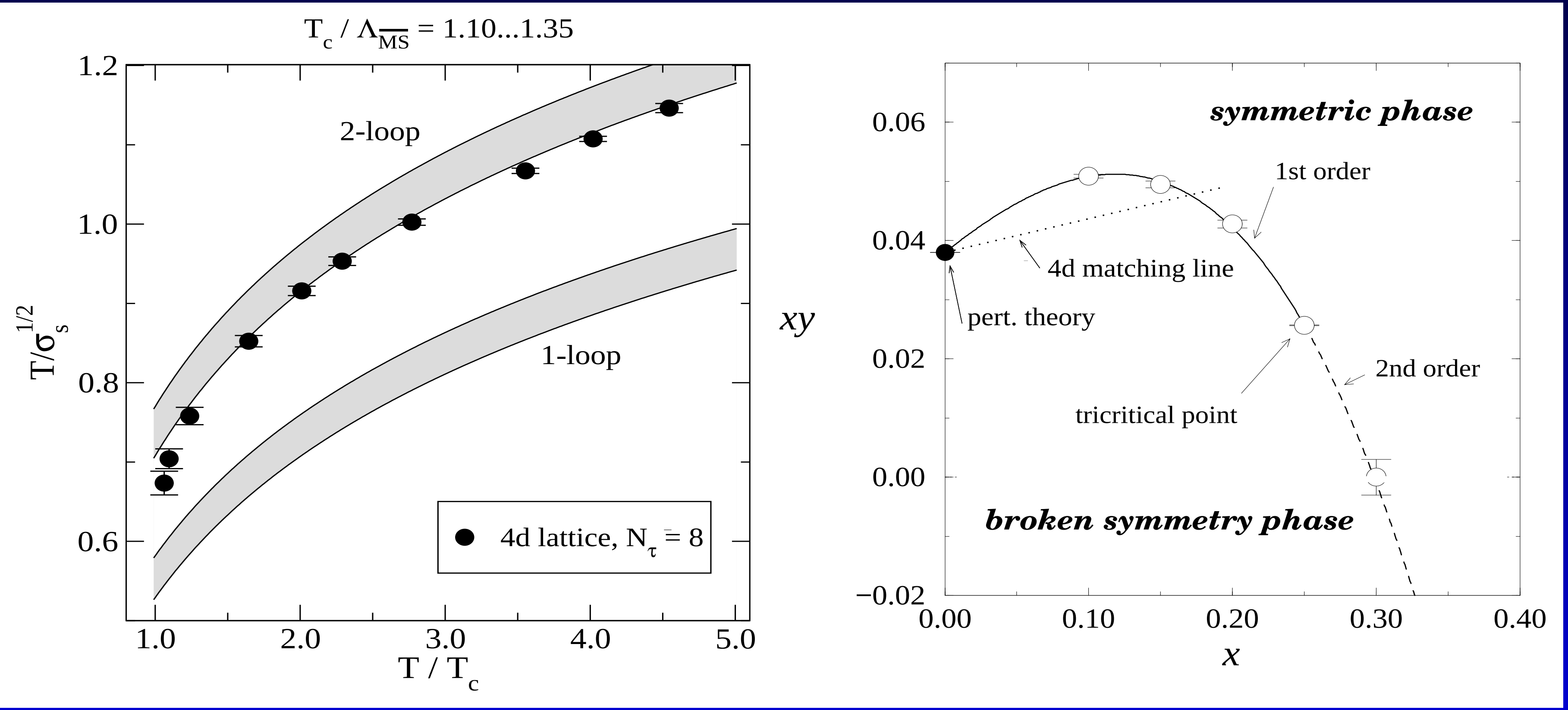

- Fundamental problem: all symmetries of original theory are *not* respected by the reduction!

The center symmetry

- Full gauge symmetry of SU(3) Yang-Mills theory

$$A_\mu(x) \to s(x)\,(A_\mu(x) + i\,\partial_\mu)\,s(x)^\dagger, s(x) \in SU(3)$$

$$s(x + \beta\,\hat{e}_t) = z\,s(x)\,, z \in Z(3)$$

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under which the Wilson line transforms as a Z(3) fundamental

$$\begin{aligned} \Omega(\mathbf{x}) &\equiv \mathcal{P}\,\exp\left[i\int_0^\beta \mathrm{d}\tau\,A_0(\tau,\mathbf{x})\right] \\ \mathrm{Tr}\,\Omega(\mathbf{x}) &\to z\,\mathrm{Tr}\,\Omega(\mathbf{x}) \end{aligned}$$

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- Ω order parameter for deconfinement transition
  - $|\langle \mathrm{Tr}\,\Omega(\mathbf{x})\rangle| = e^{-\beta\,\Delta F_q(\mathbf{x})}$

- In deconfined phase, effective potential for Ω has degenerate minima $\Omega_{\text{min}} \sim e^{i2\pi n/3}\mathbb{1}$, $n \in \{0, 1, 2\}$
  - Tunnelings between different vacua important near T_c
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- EQCD Lagrangian derived expanding effective potential around $A_0 = 0$
  - Z(3) invariance lost
  - Complex Z(3) minima $A_0 = \frac{2\pi T}{3}$ completely outside the domain of validity of eff. theory
  - (One important) cause of problems in EQCD phase diagram and predictions near T_c

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Sigma Models	
Non-linear	Linear
$\overline{\phi} \cdot \overline{\phi} = \mathbb{1}$	Polynomial V
Same long distance physics!	

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- Minimal set of dof's: A_i and Ω
  - $\Omega \in \mathrm{SU}(3) \Rightarrow$ polynomial Lagrangian non-renormalizable
- New (old) idea: replace Ω by $\mathcal{Z} \in$ GL(3,$\mathbb{C}$)
  - Coarse-grained version of Ω
  - After gauge fixing, contains $10 - 2 = 8$ unphysical dof's that are chosen heavier ($m \sim T$) than the physical ones ($m \lesssim gT$)

- Require gauge and Z(3) invariance

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- V_0 “hard”; biased towards unitary $\mathcal{Z}$
- V_1 “soft”; lifts degeneracy and ensures high-T matching to EQCD

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V(\mathcal{Z}) &= \boxed{V_0(\mathcal{Z})} + g_3^2\, V_1(\mathcal{Z}) \\
V_0(\mathcal{Z}) &= c_1\, \mathrm{Tr}\big[\mathcal{Z}^\dagger \mathcal{Z}\big] + c_2 \left(\det\big[\mathcal{Z}\big] + \det\big[\mathcal{Z}^\dagger\big]\right) \\
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- With $c_2 < 0 < c_3$ and $c_2^2 > 9c_1 c_3$, V_0 minimized by $Z = \frac{1}{3} v\Omega$ with $\Omega \in$ SU(3) and

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- Invariant under extra SU(3)×SU(3) symmetry $\mathcal{Z}(\mathbf{x}) \to A\mathcal{Z}(\mathbf{x})B,\ \ A, B \in \mathrm{SU(3)}$

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V(\mathcal{Z}) &= V_0(\mathcal{Z}) + \boxed{g_3^2\, V_1(\mathcal{Z})} \\
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- Vital for high-T matching to EQCD
- Assuming $\tilde{c}_3 > 0$ and $\tilde{c}_2^2 < \tilde{c}_1\tilde{c}_3$, V_1 minimized by $M = 0$, *i.e.* $\mathcal{Z} = \frac{1}{3}\, L(\mathbf{x})\mathbb{1}$

- $V(\mathcal{Z})$ minimized by
  - $c_2^2 > 9c_1c_3$: $\mathcal{Z} = \frac{v}{3}\, e^{2\pi i n/3}\mathbb{1}$
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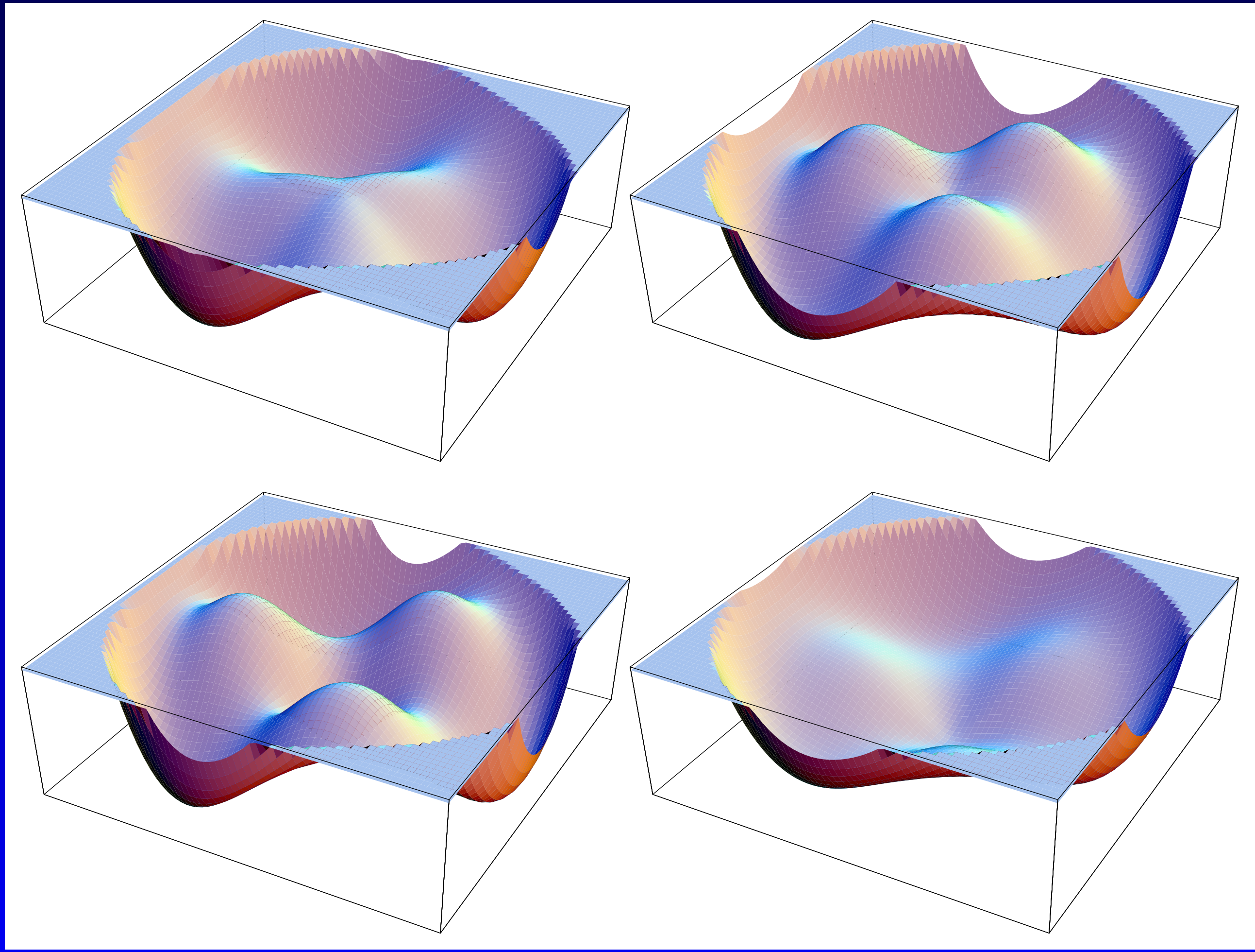

Matching to EQCD

- To determine parameters c_i and $\tilde{c}_i$, consider fluctuations around non-trivial Z(3) minima

$$\mathcal{Z} = e^{2\pi i n/3} \left\{ \frac{1}{3} v \, \mathbb{1} + g_3 \left[\frac{1}{\sqrt{6}} \left(\phi + i\chi \right) \mathbb{1} + \left(h + ia \right) \right] \right\}$$

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and write the Lagrangian in terms of the shifted fields

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- ϕ, χ and h heavy fields, with masses giving c_i's

$$\begin{aligned} c_1 &= \tfrac{1}{6}(m_\chi^2 - 3m_\phi^2)\,,\;\; c_2 = -m_\chi^2/v\,, \\ c_3 &= \tfrac{3}{4}(m_\chi^2 + 3m_\phi^2)/v^2\,,\;\; m_h^2 = m_\chi^2 + m_\phi^2 \end{aligned}$$

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- Leading order parameters solely from V_1
    - SU(3)×SU(3) invariance guarantees V_0 does not contribute
    - Result: $\tilde{c}_1 = T + \mathcal{O}(g_3^2)$, $\tilde{c}_3 = \frac{3}{4\pi^2 T} + \mathcal{O}\left(\frac{g_3^2}{T^2}\right)$

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- $\tilde{c}_2$ and v undetermined, but not needed to ensure new theory reproduces *all* EQCD predictions

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- In effective theory, end up minimizing an energy functional expressed in terms of the phases of the eigenvalues of $\mathcal{Z}$

$$F_{\rm dw}[\alpha,\beta] \equiv F_{\rm grad} + F_{\rm soft} + F_{\rm fluc} =$$

$$g_3^{-1}\,(\pi\bar{v}\,T)^2\,(\tfrac{2}{3}\sqrt{T})^3 \int_{-\infty}^{\infty} d\bar{z}\,\left[(\alpha')^2 + 3(\beta')^2 + U_1 + U_2\right]$$

with $\bar{z} \equiv g_3\sqrt{T}z$, $\bar{v} \equiv \frac{v}{T}$

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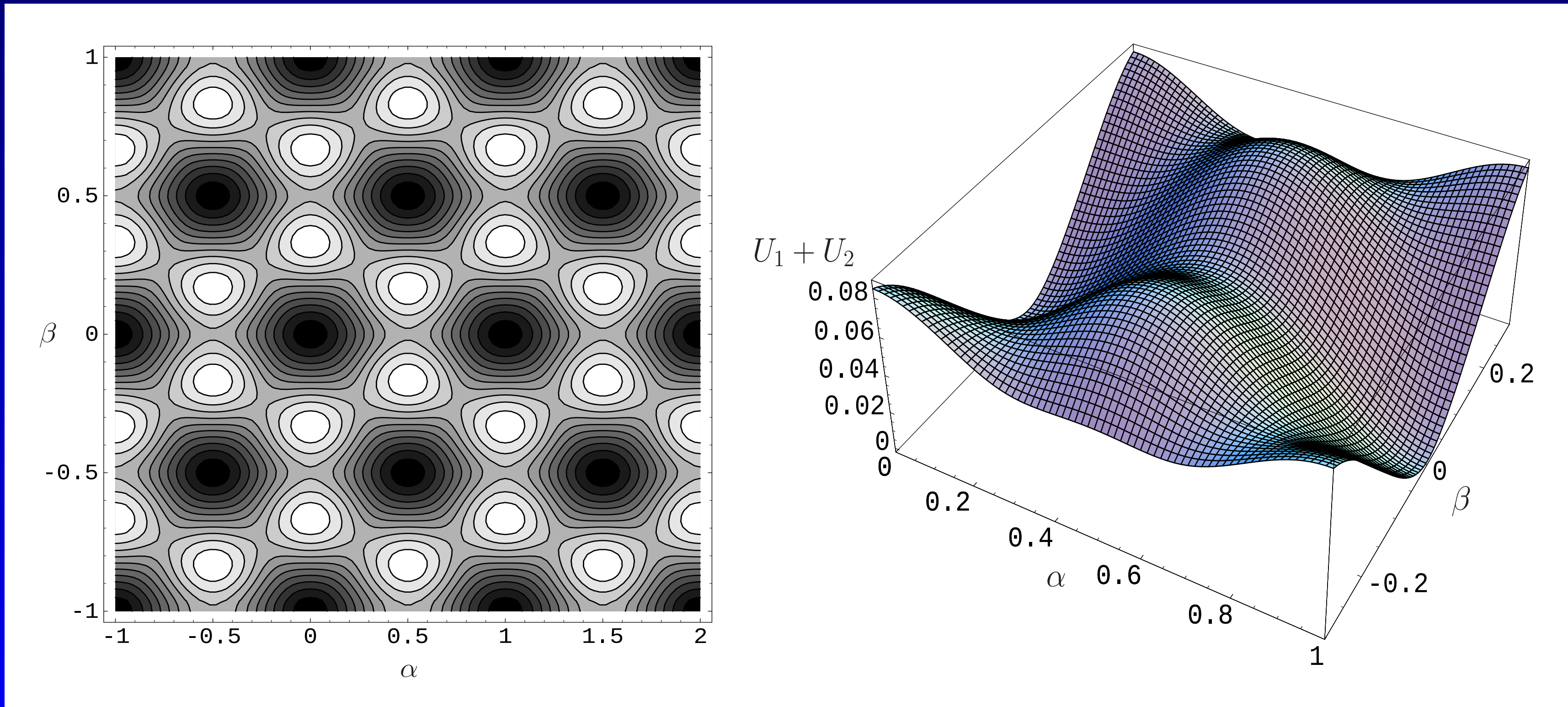

- Solve for α, β demanding that domain wall tension and width agree with full theory values
  - $\sigma_{\mathrm{YM}} = \frac{8\pi^2}{9}\,\frac{T^3}{g(T)}\,,\ \Delta z_{\mathrm{YM}} = \frac{\ln(4)-1/2}{g(T)\,T}$

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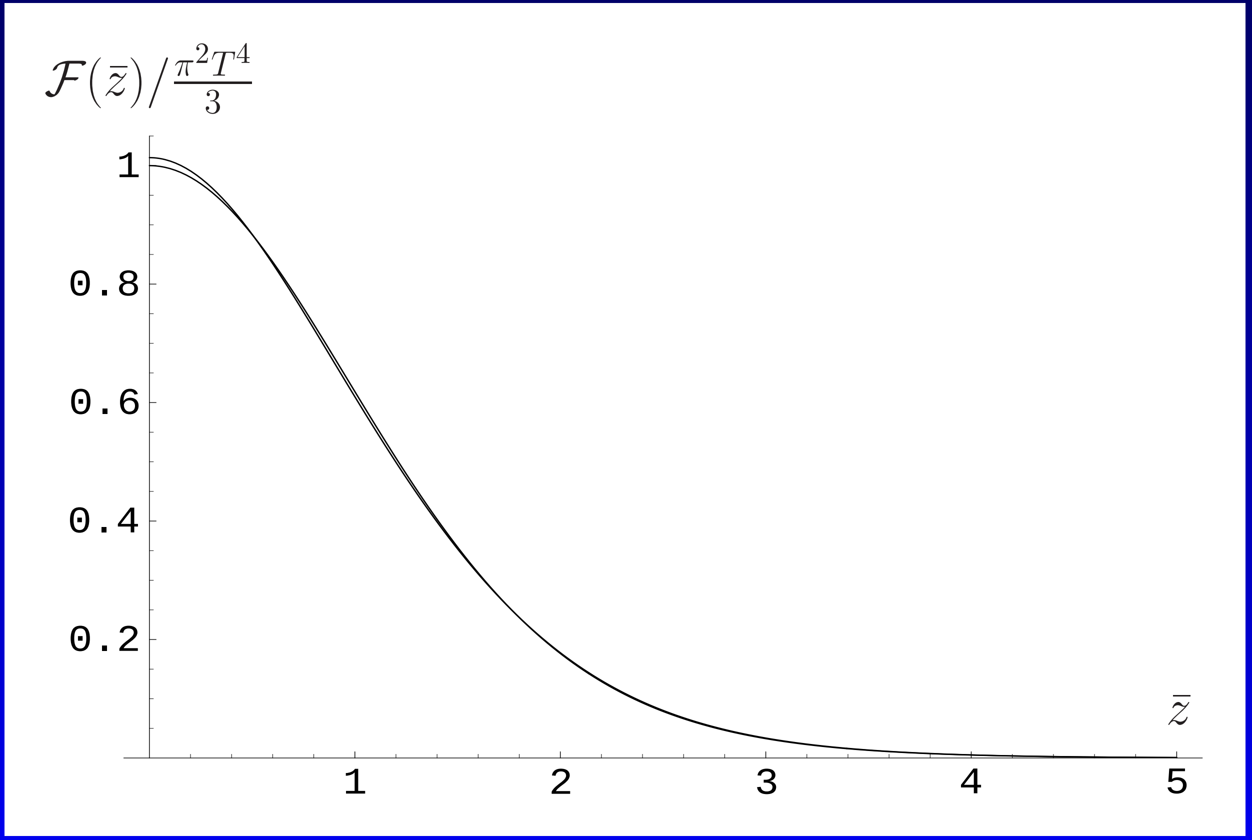
  - $\sigma_{\rm YM} = \frac{8\pi^2}{9}\,\frac{T^3}{g(T)}\,,\; \Delta z_{\rm YM} = \frac{\ln(4)-1/2}{g(T)\,T}$
- Result: $v/T = 3.005868,\;\; \tilde{c}_2 = 0.118914$



Phase diagram of new theory

- Without perturbative matching, phase diagram parametrized by 6 dimensionless constants
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  - At weak coupling, strongly 1st order transition at $m_\phi/m_\chi = 1/3$

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Phase diagram of new theory

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- In full theory, phase transition known to be weakly 1st order $\Rightarrow$ latter scenario favored

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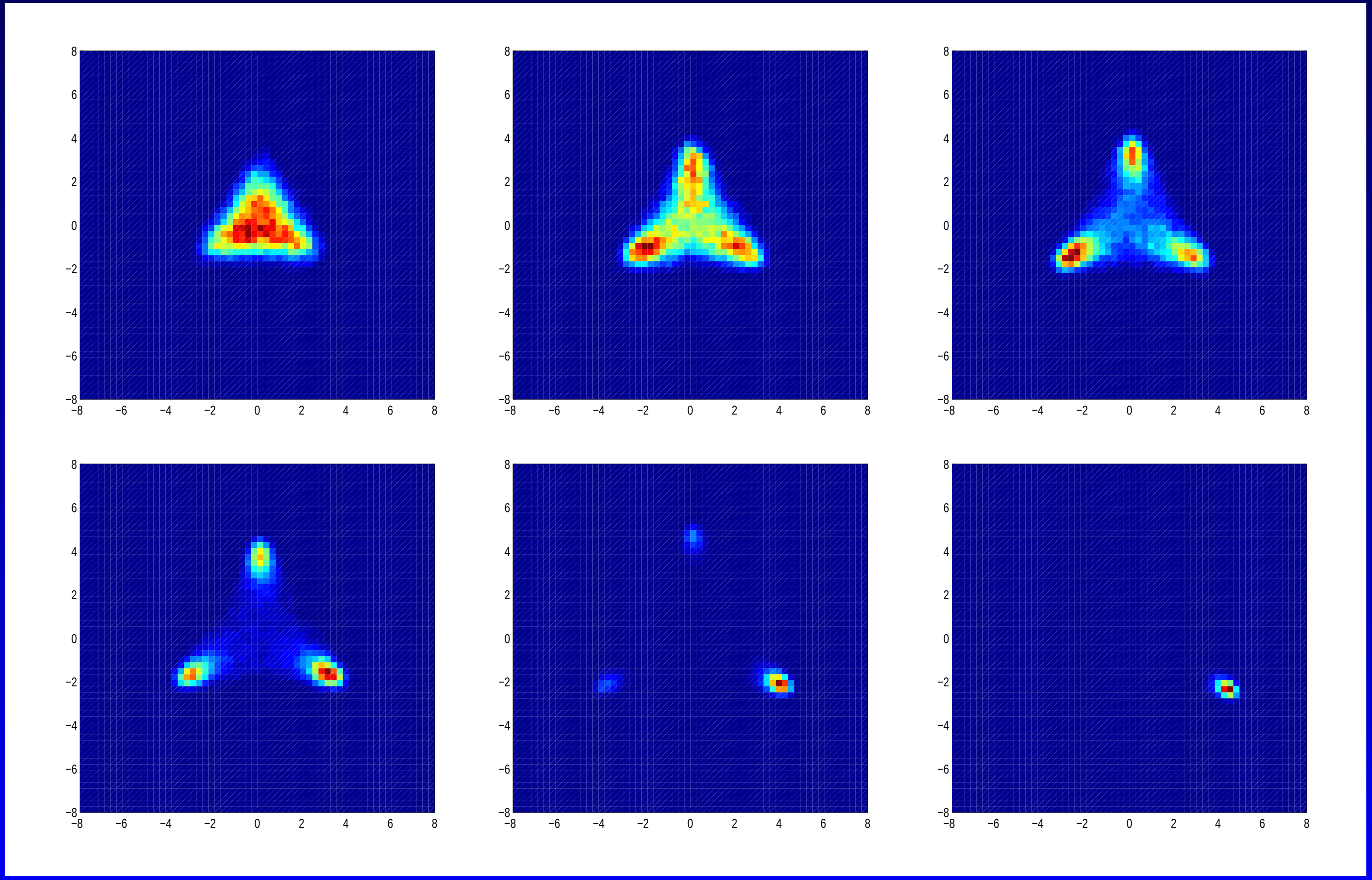
  - Simulations underway (Kajantie, Kurkela)



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- Possible generalizations: addition of quarks through soft Z(3) breaking terms, higher N_c, ...